The Geometric Distribution

www.farmermaths.com

August 20, 2022

Hi boys!

This sheet marks the beginning of Farmer Statistics, a series which I am making in remembrance to **G the Great**.

The Geometric Distribution is used to model the number of Bernoulli trials required to achieve a success given the probability of each individual trial p. The assumptions made are same as that of the Binomial Distribution.

$$X \sim Geo(p) \Rightarrow P(X = x) = (1-p)^{x-1} \cdot p$$

$$X \sim Geo(p) \Rightarrow P(X \leq x) = \sum_{n=1}^{n=x} ((1-p)^{n-1} \cdot p)$$

That's all the time we have for explanations today.

1 The Farmer's Easy Problems

1) a) The probability that a certain rabbit playing Snooker pots any given red ball is 0.04. Find the probability of him being successful on the 25th try.

b) The probability that he pots any given coloured ball after potting a red ball is 0.36. Find the probability of him potting the 11th and 12th balls, given that the 1st ball is red. (Alternates between coloured and red)

c) Now find the probability of him potting the 97th, 98th and 99th balls.

2) A botanist is planting crops. Every time he gets a seed less massive than 10 grams, he throws it away and tries the next. $X \sim N(5, 4^2)$ where X is the mass of a given seed in grams. What is the probability that he uses the 15th seed he takes?

3) The botanist is severely addicted to alcohol. He decides to quit it completely to try and end his reliance. On any given day during this period, the probability that he will relapse and have alcohol again is 0.07. Find the probability of the botanist withstanding more than 30 days without relapsing on alcohol.

2 Worked Solutions

1) a) $LetX \sim Geo(0.04)$
$P(X = 25) = 0.96^{24} \cdot 0.04 = 0.0150$

b) $0.04 \cdot 0.36 = 0.0144$
$LetX \sim Geo(0.0144)$
$\frac{12}{2} = 6$
$P(X = 6) = (1 - 0.0144)^5 \cdot 0.0144 = 0.01339$

c) $0.04 \cdot 0.36 \cdot 0.04 = 0.000576$
$LetX \sim Geo(0.000576)$
$\frac{99}{3} = 33$
$P(X = 33) = (1 - 0.000576)^{32} \cdot 0.000576 = 0.0005655$

2)$P(X \geq 10) = 0.105650$
$LetY \sim Geo(0.105650)$
$P(Y = 15) = (1 - 0.105650)^{14} \cdot 0.105650 = 0.02213$

3)$LetX \sim Geo(0.07)$
$P(X \leq 30) = \sum_{n=1}^{n=30}((1 - 0.07)^{n-1} \cdot 0.07) = 0.886633$
$P(X > 30) = 1 - P(X \leq 30) = 1 - 0.886633 = 0.1134$