Farmer Olympiad 2022 Round II

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You have 30 minutes to complete all 5 questions. Give answers on a separate sheet.

1. Which one of the following shapes is the odd one out?

   A. Coffee Mug   B. Circle   C. Pyramid   D. Doughnut   E. Triangle

2. A botanist is trying to understand the equation $x^4 + 24x^2 + y + 16 = y^3 + 8x^3 + y^2 + 32x$.

   (a) He first draws the curve given by the equation. Determine whether the curve goes through the origin, and hence or otherwise, find the points at which the curve intersects the x and y-axes.

   (b) Find points such that $x \geq 1.4$ at which the curve intersects the curve given by the quintic equation $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32 = 0$

3. An n-th degree polynomial function $f(x)$ is such that $f(4) = f(8)$. What can be deduced about the shape of $f(x)$?

4. Let every 49th second of a minute be a grand second. Let every 49th minute of an hour be a grand minute.

   (a) What is the smallest angle between the second and minute hands of a clock at the grand second of a grand minute?

   (b) Let a grand day be a date in the form $YYYY - MM - DD$ where the sum of each digit is 49. What is the earliest date for a grand day after the start of the year 2000?

   (c) Let a grand time be a date and a time in the form $YYYY - MM - DD - hh - mm - ss$ where the sum of each digit is 49. What is the earliest date and time for a grand time after the start of the year 2000?

5. Prove or disprove whether the following expressions converge.

   (a) $\sum_{n=1}^{\infty} \dfrac{kx^n}{n!}$

   (b)   i. $\int_0^{\infty} e^{-st}(sin^2(t) - 3cos^3(t) + 7t^3)dt$

      ii. Without evaluating part (i), explain what this type of mapping could be used for.

   (c)   i. $\sum_{x=1}^{\infty} \dfrac{2(1+5x)^5 - 6250(1+x)^5 + 25000(1+x)^4 - 39998(1+x)^3 + 31994x^2 + 51194x + 21246}{x^5}$

      ii. Evaluate the expression in part (i).