

Farmer Olympiad 2022 Round 2 Answers

1. C. The coffee mug is topologically equivalent to the doughnut and the circle is topologically equivalent to the triangle, leaving the pyramid as the odd one out.
2. (a) Substituting $x = 0, y = 0$ into the equation gives $16 = 0$, which is inconsistent. Hence we can determine that the curve does not go through the origin.
 (b) $(2, 0)$, can be found by substitution.
3. There lies a stationary point such that $x \in (4, 8)$ by Rolle's Theorem.
4. (a) 0, by inspection
 (b) $2999 - 09 - 29$
 (c) $2000 - 01 - 08 - 19 - 59 - 59$
5. (a) By the ratio test for convergence and k and x being constants,

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{x}{n+1} = 0$$

Hence the sum of the sequence converges. This proof can also be done by pointing out that the rate of growth of $n!$ is greater than that of x^n i.e. $e^{n \ln x}$.

- (b) i. All terms have rate of growth less than that of e^x . Where $\mathcal{L}\{f(t)\}(s) = \int_0^\infty e^{-st} f(t) dt$ by the linearity of this integral, i.e. $\mathcal{L}\{\sin^2(t) - 3\cos^3(t) + 7t^3\} = \mathcal{L}\{\sin^2(t)\} - 3\mathcal{L}\{\cos^3(t)\} + 7\mathcal{L}\{t^3\}$, which all tend towards zero as t approaches infinity.
- ii. This integral transform, viz. the Laplace Transform is used to map functions of t and their function spaces to that of a complex variable s .
- iii. With simple arithmetic it can be found by $2 \cdot 5^5 = 6250$ and $(2 \cdot 5 \cdot 5^4) - (6250 \cdot 5) = 0$ that all terms in the numerator of x^5 and x^4 are cancelled out. However, by $40000 - 39998 = 2$ we find that there remains the term $2x^3$. We can also find that all other terms are removed by the non-binomial-expansion terms that follow. This leaves us having to find $\sum_{x=1}^\infty \frac{2x^3}{x^5} = 2 \sum_{x=1}^\infty \frac{1}{x^2}$.
 Knowing that $\sum_{x=1}^\infty \frac{1}{x^2} = \frac{\pi^2}{6}$, we find the answer being $\sum_{x=1}^\infty \frac{1}{x^2} = \frac{\pi^2}{3}$.